Calculation and Measurement of the Noise Figure of a Maser Amplifier*

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Summary—**The noise performance of regenerative amplifiers is reviewed and equations are obtained which serve to interpret a measurement of noise from an ammonia molecular beam maser amplifier.**

The measurement is accomplished by means of a double heterodyne system in which a detuned maser oscillator serves as second local oscillator. The measured noise figure is 3.5±0.5 db, as predicted by theory for the slightly undercoupled circuit used. Although no beam noise is observed, the experimental uncertainty places an upper limit of 40°K on the spontaneous emission noise temperature of the ammonia beam.

Introduction

The most important source of excess noise in electronic microwave amplifiers is the shot noise carried by the electron stream. This source of excess noise is absent in a maser[1] in which electromagnetic radiation is obtained from uncharged atomic systems. This fact was pointed out by Gordon, *et al.*, in their original report, in which it was shown that in the absence of other noise sources the maser technique might lead to a nearly ideal amplifier. The present measurement (the results of which have already been reported briefly elsewhere[2]) was undertaken to test this supposition in the specific instance of a regenerative maser employing a molecular beam of ammonia.

General Discussion

Noise Performance of a Regenerative Amplifier

There appears to be some need for an elementary discussion of the noise performance of regenerative amplifiers because there has been a tendency among workers in this field to use various definitions of noise figure and noise temperature that differ from each other, as well as from the accepted standard. Such a variety is not especially serious in itself since a study of the particular definition can always make the specific result intelligible. It may, however, be less confusing to use the standard definitions as far as possible, even though, as we shall see presently, they are not completely adequate for the present purpose. The pertinent definitions are:[3,4]

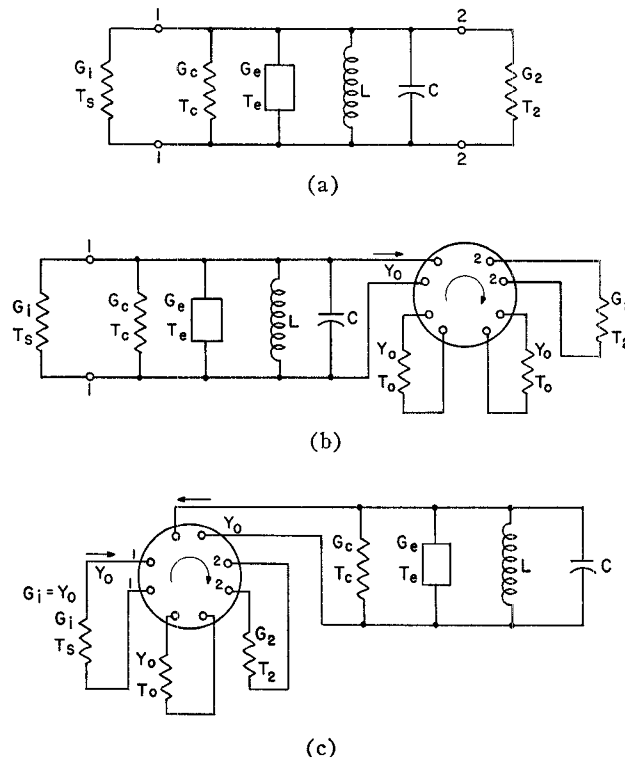

Fig. 1—(a) Transmission maser, (b) transmission maser with ideal load isolator, (c) reflection maser with ideal circulator.

Noise Figure: Of a linear system at a selected input frequency, the ratio of 1) the total noise power per unit bandwidth . . . available at the output terminals to 2) the portion thereof engendered at the input frequency by the input termination, the noise temperature of which is standard (290°K) at all frequencies.

Noise Temperature: At a pair of terminals and a specific frequency, the temperature of a passive system having an available noise power per unit bandwidth equal to that of the actual terminals.

Available Power: . . . the power that would be delivered to the output external termination . . . if the admittance of the external termination were the conjugate of the output driving-point admittance.

We shall choose as our "specific frequency" the center frequency of the ammonia resonance, and will consider the circuit configurations shown in Fig. 1, with the maser cavity used in transmission and reflection.

The circuit of Fig. 1(b) is very similar to the circuit actually used for this measurement.

The difficulty mentioned above arises in connection with the circuit configurations of Fig. 1(a), because the

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[1] J. P. Gordon, H. J. Zeiger, and C. H. Townes, "The maser—new type of microwave amplifier, frequency standard, and spectrometer," *Phys. Rev.*, vol. 99, p. 1264; 1955.

[2] J. C. Helmer, "A maser noise measurement," *Phys. Rev.*, vol. 107, pp. 902–903(L); August 1, 1957.

[3] "IRE standards on electron tubes: definitions of terms, 1950," Proc. IRE, vol. 38, pp. 426–438; April, 1950.

[4] "IRE standards on receivers: definitions of terms, 1952," Proc. IRE, vol. 40, pp. 1681–1685; December, 1952.

driving point impedance at terminals 2 may have a negative real part under a set of possible operating conditions, and thus available power is undefined. In the circuit configurations of Fig. 1(b) and 1(c) the driving point admittance at the output terminals is just the characteristic admittance Y_0 of the transmission line, so that no problem arises.

It might be pointed out here that the circuit element G_e shown in Fig. 1 represents the negative conductance of the ammonia molecular beam. It has recently been shown[5] that such a circuit representation is possible, and that an effective temperature T_e can be defined for this element, so that its available noise power is kT_e[6] watts per unit bandwidth.

We now write down the noise power per unit bandwidth at resonance flowing toward the load G_2 from all admittances G_i, G_c, G_e, Y_0 and G_2, where

$T_s = 290°K$, G_i = temperature and conductance of input coupling,
T_c, G_c = cavity temperature and conductance,
T_e, G_e = noise temperature and conductance of active medium. G_e is negative,
T_0, Y_0 = temperature and characteristic conductance of isolator or circulator,
T_2, G_2 = temperature and conductance of the load.[7]

For Fig. 1(b),

$$P_b = \frac{KT_s 4G_i Y_0 + KT_c 4G_c Y_0 + KT_e 4 \mid G_e Y_0 \mid}{(G_i + Y_0 + G_c + G_e)^2} + KT_0 \left(\frac{Y_0 - G_i - G_c - G_e}{Y_0 + G_i + G_c + G_e} \right)^2. \quad (1)$$

For Fig. 1(c),

$$P_c = KT_s \left(\frac{Y_0 - G_c - G_e}{Y_0 + G_c + G_e} \right)^2 + \frac{KT_c 4G_c Y_0 + KT_e 4 \mid G_e \mid Y_0}{(Y_0 + G_c + G_e)^2}. \quad (2)$$

A measure of the sensitivity of the amplifier of Fig. 1(a) may be obtained by defining its noise figure, F_a, so that if it is connected to a receiver of noise figure F_r, the noise figure of the combination is given by

$$F = F_a + \frac{F_r - 1}{\mu}, \quad (3)$$

where μ is the gain of the maser. In this way the definition of noise figure is tied to the way in which it is measured, as it always should be. A feature of the circuit of Fig. 1(a) is that noise generated by the load conductance G_2 is amplified by and reflected from the maser so that it adds coherently to the generating voltage. From this noise the power generated by G_2, which contributes to the noise figure of the receiver, must be subtracted. Thus, we obtain

$$P_a = \frac{KT_s 4G_i G_2 + KT_c 4G_c G_2 + KT_e 4 \mid G_e \mid G_2}{(G_i + G_2 + G_c + G_e)^2} + \frac{KT_2 4G_2^2}{(G_i + G_2 + G_c + G_e)^2} - KT_2. \quad (4)$$

In each case the power gain, μ, may be obtained by letting all temperatures, except T_s, be zero. Thus

$$\mu_a = \frac{4G_i G_2}{(G_i + G_2 + G_c + G_e)^2} \quad (5)$$

$$\mu_b = \frac{4G_i Y_0}{(G_i + Y_0 + G_c + G_e)^2} \quad (6)$$

$$\mu_c = \left(\frac{Y_0 - G_c - G_e}{Y_0 + G_c + G_e} \right)^2. \quad (7)$$

In Fig. 1(b) and 1(c) it has been assumed that G_2 is matched to Y_0. A mismatch in G_2 does not affect the signal-to-noise *ratio* at the output, but it does affect the gain. The noise figures may be immediately obtained by the formula

$$F = \frac{P}{\mu K T_s}. \quad (8)$$

Thus

$$F_a = 1 + \frac{T_c G_c + T_e \mid G_e \mid + T_2 G_2}{T_s G_i} - \frac{1}{\mu_a} \cdot \frac{T_2}{T_s} \quad (9)$$

$$F_b = 1 + \frac{T_c G_c + T_e \mid G_e \mid + T_0 Y_0}{T_s G_i} + \frac{T_0}{T_s} \left\{ \frac{1}{\mu_b} - 2 \left(\frac{1}{\mu_b} \frac{Y_0}{G_i} \right)^{1/2} \right\} \quad (10)$$

$$F_c = 1 + \frac{(\sqrt{\mu_c} + 1)^2}{\mu_c} \cdot \frac{T_c G_c + T_e \mid G_e \mid}{T_s G_i}. \quad (11)$$

Here the algebra has been arranged so that a limiting form is apparent. As the gain becomes sufficiently high, all equations have the form

$$F = \frac{\sum_x T_x \mid G_x \mid}{T_s G_i} \quad (12)$$

where the summation is taken over all conductances,

[5] M. W. Muller, "Noise in a molecular amplifier," *Phys. Rev.*, vol. 106, pp. 8–12; April 1, 1957.
R. V. Pound, "Spontaneous emission and the noise figure of maser amplifiers," *Ann. Phys.*, vol. 1, pp. 24–32; April, 1957.
K. Shimoda, H. Takahasi, and C. H. Townes, "Fluctuations in amplification of quanta with application to maser amplifiers," *J. Phys. Soc. Japan*, vol. 12, pp. 686–700; June, 1957.
M. W. P. Strandberg, "Inherent noise of quantum-mechanical amplifiers," *Phys. Rev.*, vol. 106, pp. 617–620; May 15, 1957.

[6] We use the approximation

$$\frac{\hbar\omega}{\exp(\hbar\omega/KT) - 1} = KT.$$

[7] Note the sentence following (7).

including G_s. Also, if $T_x = T_s$ for all x *except* e, then

$$F = \frac{\sum_{x \neq e} G_x}{G_i} + \frac{T_e}{T_s} \frac{|G_e|}{G_i}. \tag{13}$$

Since for high gain $G_e + \sum_{x \neq e} G_x \simeq 0$, we have

$$F \simeq \frac{Q_i}{Q_L}\left(1 + \frac{T_e}{T_s}\right), \tag{14}$$

where Q_i is the radiation Q of the input coupling and Q_L is the cold loaded Q of the cavity.

In the limit of high gain the reflection circuit of Fig. 1(c) inherently has the best noise figure because it has one less conductance to add noise to system. However, with the circuit of Fig. 1(a) it is possible to work at a lower gain, so that the amplifier is matched to the load G_2. In this case the noise generated by G_2 is not reflected by the maser and the noise figure is simply

$$F_a' = 1 + \frac{T_c G_c + T_e |G_e|}{T_s G_i}. \tag{15}$$

This type of operation may require a very small value of G_2 which, in comparison with the circuit of Fig. 1(c), results in a tendency toward much greater gain instability due to fluctuations in cavity loading.

As the gain of the circuit of Fig. 1(a) is further reduced, a noise figure less than unity becomes possible. This is not mysterious, it is the consequence of a combination of low gain and mismatch which is such that the total noise power at terminals 2, due to all sources *except* G_i, is less than KT_2. If $T_e = 0$, this condition occurs for values of gain so that

$$\mu_a < \frac{T_2 G_i}{T_c G_c + T_2 G_2}. \tag{16}$$

In the circuit of Fig. 1(b) a noise figure of less than unity is not possible, as may be seen from (1). However, the possibility still exists that the maser cavity may be matched to the isolator, so that noise generated by Y_0 is absorbed and not reflected into the output. It also is possible to cool the isolator to reduce its noise contribution.

A quantity that is frequently quoted as a measure of the sensitivity of a maser amplifier under the improper designation "noise temperature" is the excess noise temperature per unit gain of the amplifier, or

$$\text{"Noise Temperature"} = 290° \times (F - 1). \tag{17}$$

This quantity would be the true noise temperature if $\mu = 1$ and $T_i = 0°$K.

Noise Temperature of the Ammonia Beam

We have regarded the ammonia beam in the maser as a negative conductance G_e which emits noise power kT_e per unit bandwidth. It has been shown elsewhere[5] that the noise temperature T_e can be defined in terms of the populations of the two quantum states of the ammonia which participate in the interaction:

$$\exp(\hbar\omega/kT_e) = n_+/n_- \tag{18}$$

where n_+ and n_- are the populations of the upper and lower energy levels, respectively.

Since it is thought that the separating action of the inhomogeneous focusing field in an ammonia maser is quite efficient, one may expect the entering beam to consist almost entirely of upper-state molecules and thus to have a temperature near 0°K. Since $|G_e|$ is of the same order of magnitude as G_c and G_i, one thus would expect its contribution to the noise to be quite negligible, unless the amplifier were cooled to very low temperatures. It is indeed true that this noise contribution has not yet been observed.

Nevertheless, it should be pointed out that T_e might be substantially larger than the value one would compute from the population ratio of the entering beam. This is so because the populations n_+ and n_- should include all the molecules in these states in the cavity, including the "thermalized" molecules that have collided with the cavity walls or that have diffused into the cavity from the circumambient atmosphere. These molecules, of course, will contribute to n_- much more heavily than the entering beam and thus will tend to raise T_e.

It is very difficult to make an estimate of this contribution which depends on a number of parameters some of which are only very sketchily known, such as transverse velocities in the beam, the partial pressure of NH_3 in the vacuum envelope, and the flux of ammonia molecules into the cavity. We have made some estimates based on rough guesses of some of these quantities which indicate that the noise contribution of the ammonia might be between 0.1 and 0.5 db, corresponding to T_e between roughly 7°K and 35°K.

The Measurement

The problem of measuring the noise figure of an ammonia beam amplifier is essentially the problem of overcoming the noise output of a K-band microwave receiver with a very narrow band noise signal. It is instructive to examine the conditions which must be met in order to obtain a ratio of signal noise to receiver noise of unity. Two cases arise. In one case the receiver bandwidth is greater than the maser bandwidth. Assuming a maser noise figure of 1, the necessary condition for unity signal to noise ratio is

$$\frac{\mu B_m}{(F_r - 1)B_r} = 1 \tag{19}$$

where

μ = maser power gain,
B_m = maser bandwidth,
F_r = receiver noise figure,
B_r = receiver bandwidth,
$B_r > B_m$.

In the other case, the receiver bandwidth is less than the maser bandwidth and the necessary condition is simply

$$\frac{\mu}{F_r - 1} = 1. \tag{20}$$

For unity gain, the maser bandwidth is about 4000 cycles and it decreases in inverse proportion to the square root of the power gain of the maser. Thus, a maser with a gain of 20 db would have a bandwidth of about 400 cycles. On the other hand, K-band receivers typically have noise figures on the order of 20 db, primarily due to the crystal conversion loss, and have IF bandwidths on the order of 2 mc. Narrower IF bandwidths are difficult to use because of the stability requirements on the klystron local oscillator. For a 2-mc IF bandwidth, a local oscillator stability of 1 part in 10^5 is required, in order to keep received signals in the central portion of the IF amplifier response characteristic.

According to (19), we now see that fantastic maser gains are required in order to overcome the noise in the receiver. Such a measurement has been achieved by Alsop, *et al.*,[8] using a super-regenerative maser in order to obtain the required gain. A somewhat different approach has been used at Bell Telephone Laboratories.[9] Here, a maser preamplifier is attached to the front end of the receiver in order to lower the noise figure of the system. This requires the use of a circulator to which a second maser, whose noise figure is to be measured, is connected. In addition, modulation of the maser output and a lock-in detection system are employed, in order to reduce the noise bandwidth of the system.

Still a third solution to the problem has been tried by the authors. Here conditions are created such that (20) applies. The receiver shown in Fig. 2 is a double superheterodyne receiver with two mixers and two IF strips. The first mixer is driven by a klystron local oscillator and this is followed by a 2-mc wide IF strip. The second mixer works into a 50-cycle bandwidth audio strip. Ordinarily, the second local oscillator would be attached to the second mixer at this point. However, in order to keep the signal in the audio strip, the second local oscillator would have to follow the instability of the first. In other words, a receiver stability of about 1 part in 10^{10} is required in order to keep the signal centered in the pass band of the audio amplifier. The solution to this problem is to introduce a signal from a maser oscillator at the front end of the receiver along with the noise signal from the maser amplifier. These two signals are amplified independently in the first IF strip. Since they are both converted by the same local oscillator, the frequency difference between the two signals remains constant.

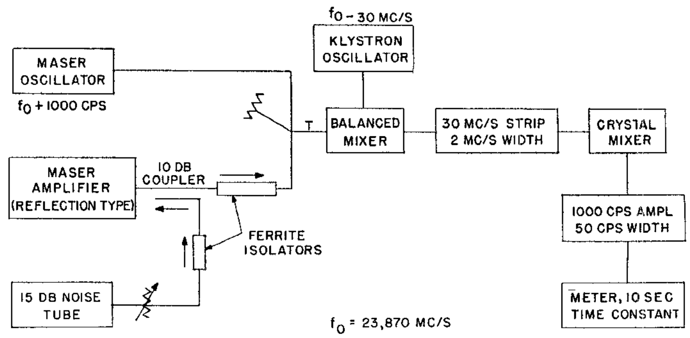

Fig. 2—Block diagram of apparatus.

When the signal from the maser oscillator reaches the second mixer, it has an amplitude of a few volts and is capable of acting as a local oscillator. This local oscillator signal then converts the amplifier signal, which is originally detuned from it by 1000 cycles, so that it is amplified in the audio strip. Provided that the maser oscillator signal is strong enough, this double superheterodyne system will be linear and will have a bandwidth equal to the bandwidth of the audio amplifier. How strong does the maser signal have to be? With a receiver noise figure of 20 db, the effective noise power input to the receiver is about 10^{-12} watts. The available maser power, however, is about 10^{-10} watts, thus giving 20-db margin over the receiver noise. This is sufficient to ensure that the noise products generated by the maser oscillator in the second mixer override all other noise contributions.

The output of the audio amplifier is rectified and its average is determined by a meter. It is unfortunate that the narrower the bandwidth of the audio amplifier the greater will be the fluctuations in the reading of the output meter for a given meter time constant. This situation has been analyzed by Dicke,[10] and one finds that the percentage fluctuation of the total output is given by

$$\frac{\Delta E}{E} \simeq \frac{1}{\sqrt{B\tau}}$$

where B = bandwidth of audio amplifier and τ = meter time constant. With a meter time constant of 10 seconds, a 4 per cent fluctuation is obtained. Since part of this fluctuation is due just to receiver noise, the fluctuation in the computed value of the maser noise is greater than 4 per cent. A longer meter time constant is not useful because of instabilities in the maser gain. However it should be pointed out that n measurements with a meter of time constant τ are equivalent to a single measurement with a meter of time constant $n\tau$. In principle, by taking a sufficient number of measurements one can obtain a noise figure to any desired degree of accuracy. Thus the ultimate accuracy is limited only by the patience of the observer and the uncertainty in

[8] L. E. Alsop, J. A. Giordmaine, C. H. Townes, and T. C. Wang, "Measurement of noise in a maser amplifier," *Phys. Rev.*, vol. 107, pp. 1450-1451(L); September 1, 1957.

[9] J. P. Gordon and L. D. White, "Experimental determination of the noise figure of an ammonia maser," *Phys. Rev.*, vol. 107, p. 1728(L); September 15, 1957.

[10] R. H. Dicke, "Measurement of thermal radiation at microwave frequencies," *Rev. Sci. Inst.*, vol. 17, pp. 268-275; July, 1946.

the calibration of the noise standard and the microwave components.

Referring again to Fig. 2 when the maser amplifier is turned on the noise power output of the receiver rises by an amount P_1. Then a known amount of noise ΔP per unit frequency interval is introduced through a calibrated directional coupler and is amplified by the maser, thereby causing the receiver noise power output to increase again by an amount P_2. The maser noise figure is then given by

$$F = \frac{\Delta P}{kT}\frac{P_1}{P_2} + \frac{1}{\mu}\,. \tag{21}$$

The results of a number of such measurements as a function of amplifier gain are shown in Fig. 3. Excluding the two high measurements at low gain, the average value of the set is $3\frac{1}{2}$ db with a standard deviation of $\frac{1}{2}$ db. A measurement of the cavity coupling shows that the cavity, being slightly undercoupled, has a circuit noise figure of $3\frac{1}{2}$ db as given by (14) with $T_e = 0$. Therefore, it is concluded that no spontaneous emission noise from the beam is observed. However, from the standard deviation of the measurements it is possible to set an upper limit to the beam noise. We conclude that the beam temperature is less than 40°K.

The high noise figure measurements at low gain can be caused by interaction between the maser oscillator and maser amplifier arising from a relatively large amplifier bandwidth at low gain. It is also true, as shown by (11), that the noise figure of the reflection amplifier rises at low gain. Since the accuracy of the gain measurements is somewhat uncertain, quantitative information is taken from the high-gain region of constant noise figure.

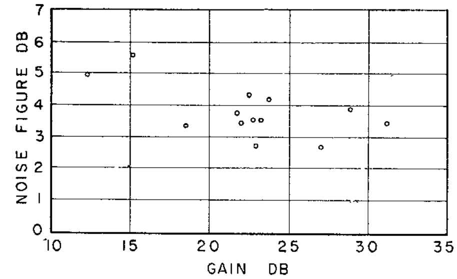

Fig. 3—Noise figure measurement.

Theoretically, we expect that the effective beam temperature can be very low. If so, the beam radiation will not be easy to detect, but it may be possible with sufficiently refined techniques. By using low-noise maser preamplifiers and by cooling the cavity and its loads, we may eventually measure spontaneous emission.

Acknowledgment

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